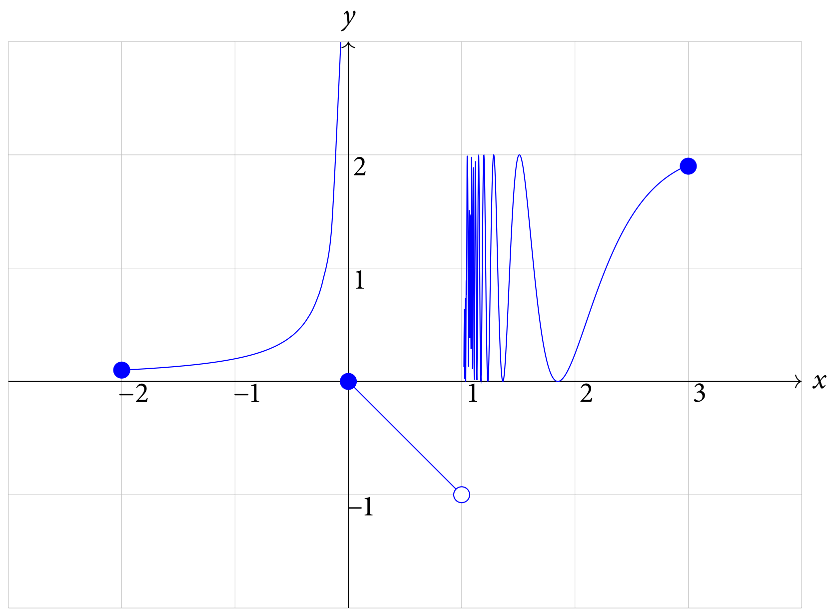
Math 1A: Calculus | **Instructor: Alexander Paulin**

Quiz 4: Limits and Continuity

Your name: *Discussions 201, 203 // 2018-09-21*

Problem 1 (5 points). The graph of $y = f(x)$ is shown below. (You may recognize it from the previous quiz!)



Specify the intervals on which f is continuous.

Solution: The function f is continuous on the intervals $[-2,0)$, $[0,1)$, and $(1,3]$. Note that it is continuous on these intervals *individually*. It is *not* continuous on, say, $[-2,0) \cup [0,1) = [-2,1)$. □

Problem 2 (5 points). Find the value of a which makes the function

$$f(x) = \begin{cases} ax - 2 & \text{if } x < \pi \\ \cos(x) & \text{if } x \geq \pi \end{cases}$$

continuous everywhere, and explain why your choice of a works.

Solution: Regardless of our choice of a, the function f is continuous for all x not equal to π. We need only pick a so that f is continuous at $x = \pi$. We already know that $\lim_{x\to\pi^+} f(x) = \lim_{x\to\pi^+} \cos(x) = \cos(\pi)$ by continuity of $\cos(x)$, so we need only pick a so that

$$\lim_{x\to\pi^-} f(x) = \lim_{x\to\pi^-} (ax - 2) = f(\pi) = \cos(\pi) = -1.$$

This is met if we take $\boxed{a = 1/\pi}$. □

(Continued on back of sheet.)

Problem 3 (5 points)**.** Prove, using the precise definition of a limit, that

$$\lim_{x\to 4}(3x-12)=0.$$

Solution: Given $\epsilon > 0$, we want to pick $\delta > 0$ such that

$$0<|x-4|<\delta \implies |3x-12-0|<\epsilon.$$

Note that we always have

$$0<|x-4|<\delta \implies |3x-12| = 3|x-4| < 3\delta.$$

Hence if we pick δ so that $3\delta \le \epsilon$ for example, we would be done. Indeed, we can just take $\delta = \epsilon/3$, and then

$$0<|x-4|<\delta=\epsilon/3 \implies |3x-12|<\epsilon$$

as desired. □